# 5-6 Inequalities in One Triangle

**Mathematics Florida Standards**

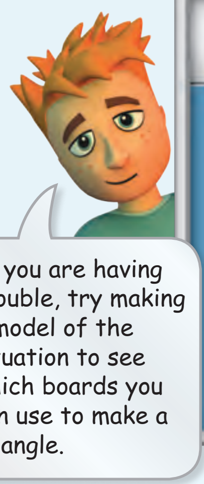
**Extends MAFS.912.G-CO.3.10** Prove theorems about triangles . . .

**MP 1, MP 3**

**Objective** To use inequalities involving angles and sides of triangles

**SOLVE IT!** **Getting Ready!**

For a neighborhood improvement project, you volunteer to help build a new sandbox at the town playground. You have two boards that will make up two sides of the triangular sandbox. One is 5 ft long and the other is 8 ft long. Boards come in the lengths shown. Which boards can you use for the third side of the sandbox? Explain.


**MATHEMATICAL PRACTICES** In the Solve It, you explored triangles formed by various lengths of board. You may have noticed that changing the angle formed by two sides of the sandbox changes the length of the third side.

**Essential Understanding** The angles and sides of a triangle have special relationships that involve inequalities.

**take note**

**Property** **Comparison Property of Inequality**

If $a = b + c$ and $c > 0$, then $a > b$.

## Proof Proof of the Comparison Property of Inequality

**Given:** $a = b + c, c > 0$

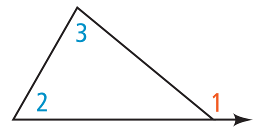
**Prove:** $a > b$

| Statements | Reasons |
|---|---|
| **1)** $c > 0$ | **1)** Given |
| **2)** $b + c > b + 0$ | **2)** Addition Property of Inequality |
| **3)** $b + c > b$ | **3)** Identity Property of Addition |
| **4)** $a = b + c$ | **4)** Given |
| **5)** $a > b$ | **5)** Substitution |

The Comparison Property of Inequality allows you to prove the following corollary to the Triangle Exterior Angle Theorem (Theorem 3-12).

take note

## Corollary   Corollary to the Triangle Exterior Angle Theorem

| Corollary | If . . . | Then . . . |
| --- | --- | --- |
| The measure of an exterior angle of a triangle is greater than the measure of each of its remote interior angles. | $\angle 1$ is an exterior angle   | $m\angle 1 > m\angle 2$ and $m\angle 1 > m\angle 3$ |

## Proof   Proof of the Corollary

**Given:** $\angle 1$ is an exterior angle of the triangle.

**Prove:** $m\angle 1 > m\angle 2$ and $m\angle 1 > m\angle 3$.

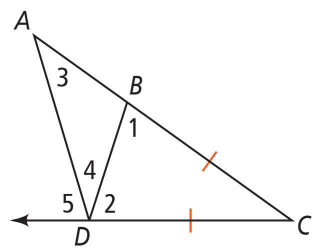
**Proof:** By the Triangle Exterior Angle Theorem, $m\angle 1 = m\angle 2 + m\angle 3$. Since $m\angle 2 > 0$ and $m\angle 3 > 0$, you can apply the Comparison Property of Inequality and conclude that $m\angle 1 > m\angle 2$ and $m\angle 1 > m\angle 3$.

## Problem 1   Applying the Corollary

**Think**

**How do you identify an exterior angle?**
An exterior angle must be formed by the extension of a side of the triangle. Here, $\angle 1$ is an exterior angle of $\triangle ABD$, but $\angle 2$ is not.

**Use the figure at the right. Why is $m\angle 2 > m\angle 3$?**



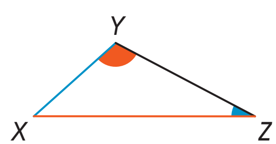

In $\triangle ACD$, $\overline{CB} \cong \overline{CD}$, so by the Isosceles Triangle Theorem, $m\angle 1 = m\angle 2$. $\angle 1$ is an exterior angle of $\triangle ABD$, so by the Corollary to the Triangle Exterior Angle Theorem, $m\angle 1 > m\angle 3$. Then $m\angle 2 > m\angle 3$ by substitution.

**Got It?** **1.** Why is $m\angle 5 > m\angle C$?

You can use the corollary to Theorem 3-12 to prove the following theorem.

take note

## Theorem 5-10

| Theorem | If . . . | Then . . . |
| --- | --- | --- |
| If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side. | $XZ > XY$   | $m\angle Y > m\angle Z$ |

*You will prove Theorem 5-10 in Exercise 40.*

**Think**

**How do you find the side opposite an angle?**
Choose an angle of a triangle. The side opposite it is the only side that does not have an endpoint at the vertex of the angle.

## Problem 2 Using Theorem 5-10

**A town park is triangular. A landscape architect wants to place a bench at the corner with the largest angle. Which two streets form the corner with the largest angle?**


Hollingsworth Road is the longest street, so it is opposite the largest angle. MLK Boulevard and Valley Road form the largest angle.

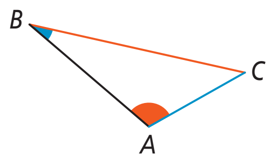
**Got It?** **2.** Suppose the landscape architect wants to place a drinking fountain at the corner with the second-largest angle. Which two streets form the corner with the second-largest angle?

Theorem 5-11 below is the converse of Theorem 5-10. The proof of Theorem 5-11 relies on indirect reasoning.

### take note — Theorem 5-11

**Theorem**
If two angles of a triangle are not congruent, then the longer side lies opposite the larger angle.

**If . . .**
$m\angle A > m\angle B$




**Then . . .**
$BC > AC$

### Proof Indirect Proof of Theorem 5-11

**Given:** $m\angle A > m\angle B$

**Prove:** $BC > AC$

**Step 1** Assume temporarily that $BC \ngtr AC$. That is, assume temporarily that either $BC < AC$ or $BC = AC$.

**Step 2** If $BC < AC$, then $m\angle A < m\angle B$ (Theorem 5-10). This contradicts the given fact that $m\angle A > m\angle B$. Therefore, $BC < AC$ must be false.

If $BC = AC$, then $m\angle A = m\angle B$ (Isosceles Triangle Theorem). This also contradicts $m\angle A > m\angle B$. Therefore, $BC = AC$ must be false.

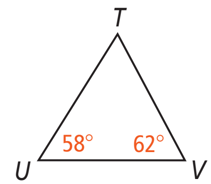
**Step 3** The temporary assumption $BC \ngtr AC$ is false, so $BC > AC$.

## Plan

**How do you use the angle measures to order the side lengths?**
List the angle measures in order from smallest to largest. Then replace the measure of each angle with the length of the side opposite.

## Problem 3 Using Theorem 5-11

**Multiple Choice** Which choice shows the sides of $\triangle TUV$ in order from shortest to longest?




Ⓐ $\overline{TV}, \overline{UV}, \overline{UT}$

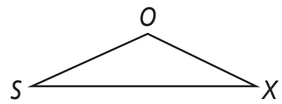
Ⓑ $\overline{UT}, \overline{UV}, \overline{TV}$

Ⓒ $\overline{UV}, \overline{UT}, \overline{TV}$

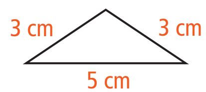
Ⓓ $\overline{TV}, \overline{UT}, \overline{UV}$

By the Triangle Angle-Sum Theorem, $m\angle T = 60$. $58 < 60 < 62$, so $m\angle U < m\angle T < m\angle V$. By Theorem 5-11, $TV < UV < UT$. Choice A is correct.

**Got It?** **3. Reasoning** In the figure at the right, $m\angle S = 24$ and $m\angle O = 130$. Which side of $\triangle SOX$ is the shortest side? Explain your reasoning.




For three segments to form a triangle, their lengths must be related in a certain way. Notice that only one of the sets of segments below can form a triangle. The sum of the smallest two lengths must be greater than the greatest length.



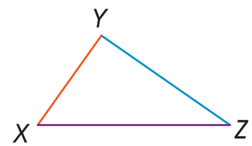


**take note**

### Theorem 5-12 Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$XY + YZ > XZ$ $\quad$ $YZ + XZ > XY$ $\quad$ $XZ + XY > YZ$




*You will prove Theorem 5-12 in Exercise 45.*

## Think

**How do you use the Triangle Inequality Theorem?**
Test each pair of side lengths. The sum of each pair must be greater than the third length.

## Problem 4 Using the Triangle Inequality Theorem

**Can a triangle have sides with the given lengths? Explain.**

**A 3 ft, 7 ft, 8 ft**

$3 + 7 > 8$ $\quad$ $7 + 8 > 3$ $\quad$ $8 + 3 > 7$

$10 > 8$ $\quad$ $15 > 3$ $\quad$ $11 > 7$

Yes. The sum of the lengths of any two sides is greater than the length of the third side.

**B 5 ft, 10 ft, 15 ft**

$5 + 10 \not> 15$

$15 \not> 15$

No. The sum of 5 and 10 is not greater than 15. This contradicts Theorem 5-12.

**Got It?** **4.** Can a triangle have sides with the given lengths? Explain.

**a.** 2 m, 6 m, and 9 m

**b.** 4 yd, 6 yd, and 9 yd

## Problem 5 Finding Possible Side Lengths

**Algebra** **In the Solve It, you explored the possible dimensions of a triangular sandbox. Two of the sides are 5 ft and 8 ft long. What is the range of possible lengths for the third side?**

| Know | Need | Plan |
| --- | --- | --- |
| The lengths of two sides of the triangle are 5 ft and 8 ft. | The range of possible lengths of the third side | Use the Triangle Inequality Theorem to write three inequalities. Use the solutions of the inequalities to determine the greatest and least possible lengths. |

Let $x$ represent the length of the third side. Use the Triangle Inequality Theorem to write three inequalities. Then solve each inequality for $x$.

| | | |
| --- | --- | --- |
| $x + 5 > 8$ | $x + 8 > 5$ | $5 + 8 > x$ |
| $x > 3$ | $x > -3$ | $x < 13$ |

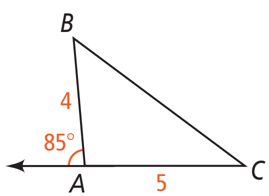
Numbers that satisfy $x > 3$ and $x > -3$ must be greater than 3. So, the third side must be greater than 3 ft and less than 13 ft.

**Got It?** **5.** A triangle has side lengths of 4 in. and 7 in. What is the range of possible lengths for the third side?

## Lesson Check

### Do you know HOW?

**Use $\triangle ABC$ for Exercises 1 and 2.**




**1.** Which side is the longest?

**2.** Which angle is the smallest?

**3.** Can a triangle have sides of lengths 4, 5, and 10? Explain.

### Do you UNDERSTAND? MATHEMATICAL PRACTICES

**4. Error Analysis** A friend tells you that she drew a triangle with perimeter 16 and one side of length 8. How do you know she made an error in her drawing?

**5. Reasoning** Is it possible to draw a right triangle with an exterior angle measuring 88? Explain your reasoning.

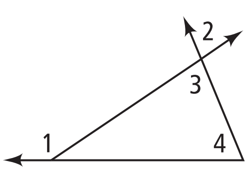
## Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

**A Practice**

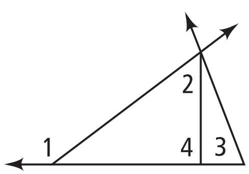
**Explain why $m\angle 1 > m\angle 2$.**

See Problem 1.

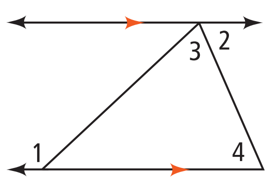
**6.**




**7.**

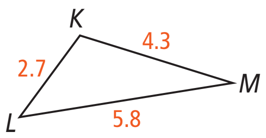



**8.**

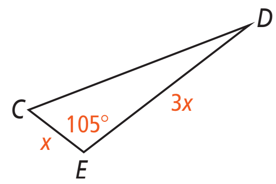


**For Exercises 9–14, list the angles of each triangle in order from smallest to largest.** See Problem 2.

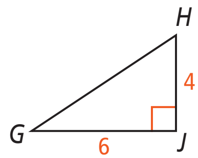
**9.**




**10.**




**11.**



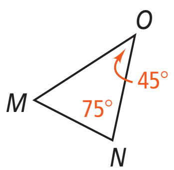

**12.** $\triangle ABC$, where $AB = 8$, $BC = 5$, and $CA = 7$

**13.** $\triangle DEF$, where $DE = 15$, $EF = 18$, and $DF = 5$

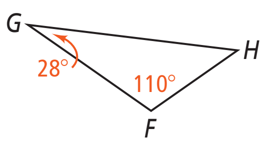
**14.** $\triangle XYZ$, where $XY = 12$, $YZ = 24$, and $ZX = 30$

**For Exercises 15–20, list the sides of each triangle in order from shortest to longest.** See Problem 3.

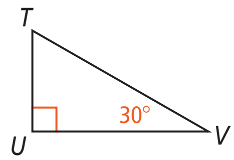
**15.**




**16.**




**17.**




**18.** $\triangle ABC$, with $m\angle A = 90$, $m\angle B = 40$, and $m\angle C = 50$

**19.** $\triangle DEF$, with $m\angle D = 20$, $m\angle E = 120$, and $m\angle F = 40$

**20.** $\triangle XYZ$, with $m\angle X = 51$, $m\angle Y = 59$, and $m\angle Z = 70$

**Can a triangle have sides with the given lengths? Explain.** See Problem 4.

**21.** 2 in., 3 in., 6 in.

**22.** 11 cm, 12 cm, 15 cm

**23.** 8 m, 10 m, 19 m

**24.** 1 cm, 15 cm, 15 cm

**25.** 2 yd, 9 yd, 10 yd

**26.** 4 m, 5 m, 9 m

**Algebra** **The lengths of two sides of a triangle are given. Find the range of possible lengths for the third side.** See Problem 5.

**27.** 8 ft, 12 ft

**28.** 5 in., 16 in.

**29.** 6 cm, 6 cm

**30.** 18 m, 23 m

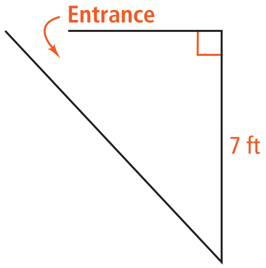
**31.** 4 yd, 7 yd

**32.** 20 km, 35 km

**B Apply**

**33. Think About a Plan** You are setting up a study area where you will do your homework each evening. It is triangular with an entrance on one side. You want to put your computer in the corner with the largest angle and a bookshelf on the longest side. Where should you place your computer? On which side should you place the bookshelf? Explain.

- What type of triangle is shown in the figure?
- Once you find the largest angle of a triangle, how do you find the longest side?



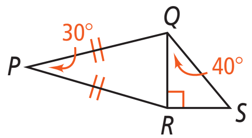

**34. Algebra** Find the longest side of $\triangle ABC$, with $m\angle A = 70$, $m\angle B = 2x - 10$, and $m\angle C = 3x + 20$.

**35. Writing** You and a friend compete in a scavenger hunt at a museum. The two of you walk from the Picasso exhibit to the Native American gallery along the dashed red line. When he sees that another team is ahead of you, your friend says, “They must have cut through the courtyard.” Explain what your friend means.

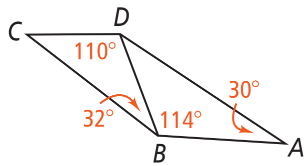

**36. Error Analysis** Your family drives across Kansas on Interstate 70. A sign reads, “Wichita 90 mi, Topeka 110 mi.” Your little brother says, “I didn’t know that it was only 20 miles from Wichita to Topeka.” Explain why the distance between the two cities does not have to be 20 mi.

**Reasoning** **Determine which segment is shortest in each diagram.**

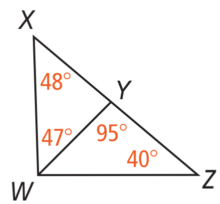
**37.**




**38.**




**39.**

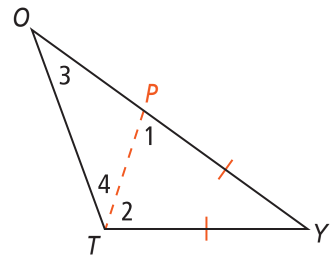



**40. Developing Proof** Fill in the blanks for a proof of Theorem 5-10: If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side.

**Given:** $\triangle TOY$, with $YO > YT$

**Prove:** **a.** ? $>$ **b.** ?

Mark $P$ on $\overline{YO}$ so that $\overline{YP} \cong \overline{YT}$. Draw $\overline{TP}$.

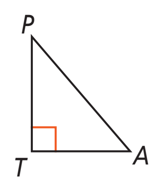



| Statements | Reasons |
|---|---|
| 1) $\overline{YP} \cong \overline{YT}$ | 1) Ruler Postulate |
| 2) $m\angle 1 = m\angle 2$ | 2) **c.** ? |
| 3) $m\angle OTY = m\angle 4 + m\angle 2$ | 3) **d.** ? |
| 4) $m\angle OTY > m\angle 2$ | 4) **e.** ? |
| 5) $m\angle OTY > m\angle 1$ | 5) **f.** ? |
| 6) $m\angle 1 > m\angle 3$ | 6) **g.** ? |
| 7) $m\angle OTY > m\angle 3$ | 7) **h.** ? |

**41.** Proof Prove this corollary to Theorem 5-11: The perpendicular segment from a point to a line is the shortest segment from the point to the line.

**Given:** $\overline{PT} \perp \overline{TA}$

**Prove:** $PA > PT$




**Challenge**

**42. Probability** A student has two straws. One is 6 cm long and the other is 9 cm long. She picks a third straw at random from a group of four straws whose lengths are 3 cm, 5 cm, 11 cm, and 15 cm. What is the probability that the straw she picks will allow her to form a triangle? Justify your answer.

**For Exercises 43 and 44, $x$ and $y$ are integers such that $1 < x < 5$ and $2 < y < 9$.**

**43.** The sides of a triangle are 5 cm, $x$ cm, and $y$ cm. List all possible $(x, y)$ pairs.

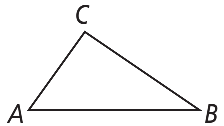
**44.** **Probability** What is the probability that you can draw an isosceles triangle that has sides 5 cm, $x$ cm, and $y$ cm, with $x$ and $y$ chosen at random?

**45.** Proof Prove the Triangle Inequality Theorem: The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

**Given:** $\triangle ABC$

**Prove:** $AC + CB > AB$

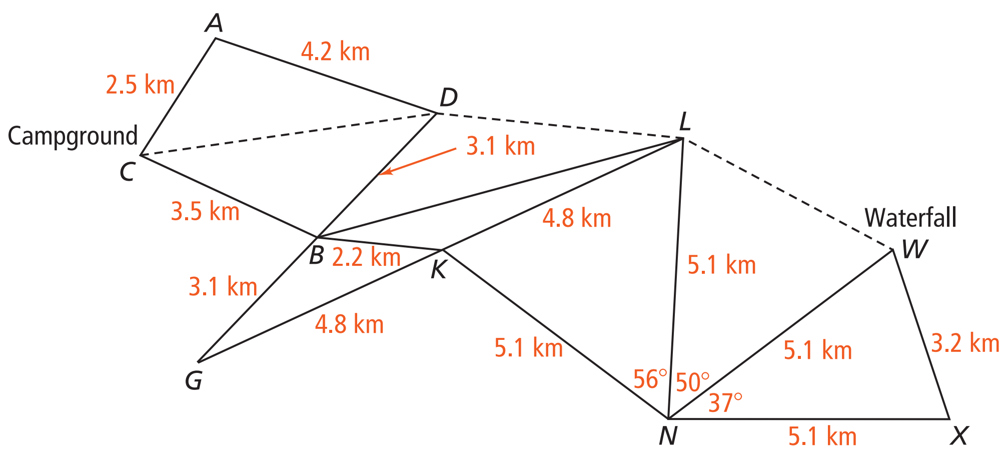
($Hint$: On $\overrightarrow{BC}$, mark a point $D$ not on $\overline{BC}$, so that $DC = AC$. Draw $\overline{DA}$ and use Theorem 5-11 with $\triangle ABD$.)




## Apply What You've Learned

MATHEMATICAL PRACTICES
MP 3

In the Apply What You've Learned in Lesson 5-1, you analyzed relationships in $\triangle DGL$ in the trail map on page 283. Now look again at the map and consider $\triangle CAD$ and $\triangle CBD$.




**a.** Using the given side lengths in $\triangle CAD$, what can you conclude about the range of possible lengths of $\overline{CD}$? Explain.

**b.** One of the hikers states that $\overline{CD}$ could be 6.6 km long. Do you agree or disagree? Give an argument to justify your response.

**c.** Using the Triangle Inequality Theorem on $\triangle CAD$ and $\triangle CBD$, determine the range of possible lengths of $\overline{CD}$.